Fractionally Quantized Hall Effect: Liquid-Crystal Ground-State and it Excitations

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It is shown that the ground-state and the lowest excited-states of two-dimensional electron system (2DES), with ion jellium background, correspond to partial crystal-like (with the period $L_x^{\square} = \sqrt{2m\pi}\ell_0$, along x; ℓ_0 is the magnetic length) correlation order among N electrons of the main region (MR; $L_x \times L_y$, $L_{x,y} \to \infty$). Many-body variational ground-state wave function of 2DES is presented at the fractional and the integral filling factors $\nu = 1/m$; $m = 2\ell + 1$ and $\ell = 0, 1, 2, \ldots$. The ground-state manifests the broken symmetry liquid-crystal state with 2DES density that is periodic along the $y-$ direction, with the period $L_x^{\square}/m$, and independent of x. At $m = 3, 5$, the ground-state has essentially lower energy per electron than the Laughlin, uniform liquid, ground-state; the same holds at $m = 1$. At $m \geq 3$, the compound form of the many-body ground-state wave function leads to the compound structure of each electron, even within the main strip (MS; $L_x^{\square} \times L_y$). These compound electrons play important role in the properties of the many-body excited-states. Obtained compound exciton (without the change of spin of the excited compound electron) and compound spin-exciton (with the change of spin of the excited compound electron) states show finite excitation gaps, for $m = 1, 3, 5$. The excited compound electron (hole) is composed, within MS, from m strongly correlated quasielectrons (quasiholes) of the total charge e/m $(-e/m)$ each; fractionally quantized at $m \geq 3$. These m quasielectrons (quasiholes) are periodically "repeated" outside MS, along x, with the period $L_x^{\square}$. The activation gap is obtained: it is given by the excitation gap of relevant compound exciton, at $m \geq 3$, and by the gap of pertinent compound spin-exciton, at $m = 1$. Quantized Hall conductance $\sigma_H = e^2/(2m\pi\hbar)$ is obtained; it is fractional at $m \geq 3$. The theory is in good agreement with experiments.

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INTRODUCTION. THEORETICAL FRAMEWORK

To date understanding is that at the filling factor $\nu = 1/3, 1/5$ the Laughlin variational wave function [1] gives the best known so far analytical approximation of exact many-body ground-state wave function. At $\nu = 1/3$, there were many attempts to obtain lower energy for ground-state than Laughlin's [1] incompressible liquid state, e.g., see references in [2]. Here we will start with the same many-electron Hamiltonian, $\hat{H}(\mathbf{r}_1, \ldots, \mathbf{r}_N)$, for 2DES of N electrons as in [1, 3], only we adopt the Landau gauge for vector potencial $\mathbf{A} = -By\hat{\mathbf{x}}$. We assume that N electrons are localized in MR of $(z = 0)-$plane; $N/L_x L_y = \nu/2\pi\ell_0^2$, where $\nu = 1/m$, $\ell_0 = \sqrt{\hbar c/|e|B}$. As in [1, 3], the classical model of neutralizing ion jellium background (IJB) is used, cf. [2]. Eigenstates $\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ of the Hamiltonian $\hat{H}(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ and their energies are determined by

$$\hat{H}(\mathbf{r}_1, \ldots, \mathbf{r}_N)\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = E_N \Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N). \quad (1)$$

We can assume that 2DES, with IJB, is located within the ribbon of width L_y bent into loop of radius $L_x/2\pi$. Then Born-Carman periodic boundary conditions $\mathbf{r}_i \pm L_x\hat{\mathbf{x}} = \mathbf{r}_i$ are holded, where $i = 1, \ldots, N$.

It is seen that the area of MR per electron, $L_x L_y/N = (L_x^{\square})^2$, where $L_x^{\square} = \sqrt{2m\pi}\ell_0$. Then the strip of the width $L_x^{\square}$, along x-direction, and of the length L_y contains $\tilde{N} =$

$L_y/L_x^{\square}$ of the (square) unit cells, $L_x^{\square} \times L_x^{\square}$. The integer number of such strips within MR is given as $n_{xs}^{\max} = L_x/L_x^{\square} = N/\tilde{N}$; for definiteness, odd (as $\tilde{N}$). Now we assume that the ground-state and, at least, the lowest excited-states of Eq. (1) correspond to partial crystal-like correlation order among N electrons of MR as

$$\mathbf{r}_{1+k\tilde{N}} = \mathbf{r}_1 + kL_x^{\square}\hat{\mathbf{x}}, \quad \mathbf{r}_{2+k\tilde{N}} = \mathbf{r}_2 + kL_x^{\square}\hat{\mathbf{x}},$$
$$\cdots, \quad \mathbf{r}_{\tilde{N}+k\tilde{N}} = \mathbf{r}_{\tilde{N}} + kL_x^{\square}\hat{\mathbf{x}}, \quad (2)$$

where $k = 0, \pm 1, \ldots, \pm(n_{xs}^{\max} - 1)/2$. Then Hamiltonian in Eq. (1) becomes: i) dependent only on $\tilde{N}$ $\mathbf{r}_i = (x_i, y_i)$ and ii) periodic, with period $L_x^{\square}$, on any x_i; $i = 1, \ldots, \tilde{N}$. For the wave function in Eq. (1): the property (i) obviously holds and it is natural to assume that the property (ii) is valid as well. Then the study within MR of Eq. (1), for 2DES of N electrons, can be reduced to the treatment of the Schrödinger equation for 2DES of $\tilde{N}$ "compound" electrons within the main strip (MS) $L_x^{\square}n_{xs}^{MS} > x_i > L_x^{\square}(n_{xs}^{MS} - 1)$, cf. Ref. [2], as

$$\hat{H}_{\tilde{N}}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}})\Psi_{\tilde{N}} = E_{\tilde{N}}\Psi_{\tilde{N}}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}}), \quad (3)$$

where, for definiteness, we assume $n_{xs}^{MS} = 0$. In Eq. (3)

$$\hat{H}_{\tilde{N}} = \hat{H}_0 + V_{ee} + V_{eb} + V_{bb}, \quad (4)$$

where the kinetic energy term

$$\hat{H}_0 = \sum_{i=1}^{\tilde{N}} \hat{h}_{0i} = \frac{1}{2m^*}\sum_{i=1}^{\tilde{N}}[\hat{\mathbf{p}}_i - \frac{e}{c}\mathbf{A}(\mathbf{r}_i)]^2, \quad (5)$$

here $\hat{\mathbf{p}} = -i\hbar\boldsymbol{\nabla}$, m^* the electron effective mass. In Eq. (4) the electron-electron potential, cf. [4]

$$V_{ee} = \frac{1}{2}\sum_{i=1}^{\tilde{N}}\sum_{j=1,j\neq i}^{\tilde{N}}\sum_{k=-N_C}^{N_C}\frac{e^2}{\varepsilon|\mathbf{r}_i - \mathbf{r}_j - kL_x^{\square}\hat{\mathbf{x}}|}$$
$$+\tilde{N}\sum_{k=1}^{N_C}\frac{e^2}{\varepsilon L_x^{\square}k}, \qquad (6)$$

physical results will not depend on $N_C \to \infty$. The electron-IJB interaction potential, cf. [3],

$$V_{eb} = -\sum_{i=1}^{\tilde{N}}\int_{MR}d\mathbf{R}\frac{e^2 n_b(\mathbf{R})}{\varepsilon|\mathbf{r}_i - \mathbf{R}|}, \qquad (7)$$

and IJB-IJB interaction term

$$V_{bb} = \frac{1}{2}\int_{MS}d\mathbf{R}\int_{MR}d\mathbf{R}'\frac{e^2 n_b(\mathbf{R})n_b(\mathbf{R}')}{\varepsilon|\mathbf{R} - \mathbf{R}'|}, \qquad (8)$$

where the subscript MR (MS) shows that integration is carried out over MR (MS); $n_b(\mathbf{R}) = const = n_b$ and $\int_{MS}d\mathbf{R}n_b = \tilde{N}$. Notice $\langle\Psi_{\tilde{N}}|\Psi_{\tilde{N}}\rangle = 1$, where integrations are over the MS, and $\epsilon = E_{\tilde{N}}/\tilde{N} = E_N/N$ is the total energy per electron.

At $\nu = 1/m$ we assume ground-state $\Psi_{\tilde{N}}^{(m)}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}})$ of Eq. (3) as [2]

$$\Psi_{\tilde{N}}^{(m)} = \sum_{n=-\ell}^{\ell}C_n(m)\Psi_{\tilde{N}}^{n,(m)}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}}), \qquad (9)$$

where $|C_n(m)|^2 = 1/m$ and

$$\Psi_{\tilde{N}}^{n,(m)} = \frac{1}{\sqrt{\tilde{N}!}}\begin{vmatrix}\varphi_{k_{x1}^{(n)}}^{(m)}(\mathbf{r}_1) & \cdots & \varphi_{k_{x1}^{(n)}}^{(m)}(\mathbf{r}_{\tilde{N}})\\ \varphi_{k_{x2}^{(n)}}^{(m)}(\mathbf{r}_1) & \cdots & \varphi_{k_{x2}^{(n)}}^{(m)}(\mathbf{r}_{\tilde{N}})\\ \vdots & \ddots & \vdots\\ \varphi_{k_{x\tilde{N}}^{(n)}}^{(m)}(\mathbf{r}_1) & \cdots & \varphi_{k_{x\tilde{N}}^{(n)}}^{(m)}(\mathbf{r}_{\tilde{N}})\end{vmatrix}, \qquad (10)$$

is the Slater determinant of single-electron wave functions (orthonormal within MS; $y_0(k_x) = \ell_0^2 k_x$)

$$\varphi_{k_{xi}^{(n)}}^{(m)}(\mathbf{r}) = e^{ik_{xi}^{(n)}x}\Psi_0(y - y_0(k_{xi}^{(n)}))/\sqrt{L_x^{\square}}, \qquad (11)$$

where $\Psi_0(y)$ is the harmonic oscillator function, $i = 1, 2, \ldots, \tilde{N}$ is the number of a unit cell within MS,

$$k_{xi}^{(n)} = \frac{2\pi m}{L_x^{\square}}\left[n_{ys}^{(i)} + \frac{n}{m}\right], \qquad (12)$$

here $n_{ys}^{(i)} = 0, \pm 1, \ldots, \pm(\tilde{N}-1)/2$, the "set" number $n = 0, \ldots, \pm\ell$; $\langle\Psi_{\tilde{N}}^{k,(m)}|\Psi_{\tilde{N}}^{n,(m)}\rangle = \delta_{k,n}$. Notice, for the ground-state Eq. (9) electron density [2],

$$n(y) = \frac{m^{-1}}{2\pi\ell_0^2}[1 + 2\sum_{k=1}^{\infty}e^{-\frac{\pi m k^2}{2}}\cos(\frac{\sqrt{2\pi m}}{\ell_0}ky)], \qquad (13)$$

depends only on y, periodically; $L_x^{\square}$ is the period. It is readily seen that the periodicity property (ii) is valid as for ground-state wave function, Eq. (9), so for $\Psi_{\tilde{N}}^{n,(m)}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}})$.

RESULTS AND DISCUSSION

Using Eq. (4) we obtain the energy $\epsilon^{(m)} = \hbar\omega_c/2 + (e^2/\varepsilon\ell_0)U(m)$ of the ground-state Eq. (9) as $< \Psi_{\tilde{N}}^{(m)}|\hat{H}_{\tilde{N}}|\Psi_{\tilde{N}}^{(m)} > /\tilde{N}$, where $U(m)$ is given by analytical expressions, deduced explicitly in [2]. After simple numerical calculations, we obtain: $U(3) = -0.42854$, $U(5) = -0.33885$, and $U(1) = -0.66510$. So $U(3)$, $U(5)$, and $U(1)$ there are substantially lower than pertinent total lowering at $\nu = 1/3$, $1/5$, and 1 for the Laughlin's variational wave function [1]: -0.4156 ± 0.0012 (-0.410 ± 0.001 is obtained in Ref. [3], by fixing relevant numerical study of [1]), -0.3340 ± 0.0028, and $-\sqrt{\pi/8} \approx -0.6267$.

We assume, for $m \geq 3$, the lowest excited state (the compound exciton) of the ground-state Eq. (9) as[2]

$$\Psi_{\tilde{N};(m)}^{i_0,j_0;\tilde{n}} = \sum_{n=-\ell}^{\ell}C_n(m)\Phi_{\tilde{N},(m);n}^{i_0,j_0;\tilde{n}}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}}), \qquad (14)$$

where the excited "partial" many-electron wave function $\Phi_{\tilde{N},(m);n}^{i_0,j_0;\tilde{n}}$ it follows from the ground-state "partial" many-electron wave function $\Psi_{\tilde{N}}^{n,(m)}$ after changing in its $\tilde{N}-$dimensional Slater determinant of the i_0-th row $\varphi_{k_{xi_0}^{(n)}}^{(m)}(\mathbf{r}_1), \varphi_{k_{xi_0}^{(n)}}^{(m)}(\mathbf{r}_2), \cdots, \varphi_{k_{xi_0}^{(n)}}^{(m)}(\mathbf{r}_{\tilde{N}})$ by the row

$$\varphi_{k_{xj_0}^{(n+\tilde{n})}}^{(m)}(\mathbf{r}_1), \varphi_{k_{xj_0}^{(n+\tilde{n})}}^{(m)}(\mathbf{r}_2), \cdots, \varphi_{k_{xj_0}^{(n+\tilde{n})}}^{(m)}(\mathbf{r}_{\tilde{N}}), \qquad (15)$$

where $\tilde{n} = \pm 1, \ldots, \pm\ell$, the implicit spin wave function is omitted. We assume that the i_0-th unit cell defined by $n_{ys}^{(i_0)}$ (where m quasihole excitations appear that constitute the compound hole) as well as the j_0-th unit cell (where m quasielectron excitations are mainly localized that constitute the excited compound electron) there are well inside of MS. For the compound exciton Eq. (14) the spin of the compound electron is not changed in a process of the excitation; excited states Eq. (14) are orthonormal and they are orthogonal to the ground-state Eq. (9). The total charge of any quasielectron (quasihole) within MS is given as e/m ($-e/m$). Due to periodic boundary conditions (in particular, the periodicity of single-electron wave functions) it is clear that the charge density of the compound exciton has its "images" along x, outside MS, with the period $L_x^{\square}$.

The study shows that at $m \geq 3$ the lowest excited state, Eq. (14), of the compound exciton (here it defines the activation gap $E_{ac}^{(m)}$) is obtained for $n_{ys}^{(j_0)} = n_{ys}^{(i_0)}$ and $\tilde{n} = \pm 1$. We obtain that dimensionless activation

gap of the compound exciton $\Delta_{ac}^{(m)} = E_{ac}^{(m)}/(e^2/\varepsilon\ell_0)$ is given, for $m = 3$, 5, as $\Delta_{ac}^{(3)} \approx 0.1016$, $\Delta_{ac}^{(5)} \approx 0.0257$. Further, the study shows [2] that at $m = 1$ the compound spin-exciton (with the change of the spin of the excited compound electron) gives the activation gap as $E_{ac}^{(1)} = |g_0|\mu_B B + 1.1843 e^2/\varepsilon\ell_0$, where many-body contribution is a bit smaller than relevant result of Hartree-Fock approximation $\sqrt{\pi/2}(e^2/\varepsilon\ell_0)$; g_0 is the bare g-factor.

Assuming that the Fermi level is located within the finite energy gap between the ground-state, Eq. (9), and excited-states, we calculate [2] (in particular, from the Kubo formula) that the Hall conductance $\sigma_H = -\sigma_{xy} = e^2/(2m\pi\hbar)$. I.e., for $m = 3, 5, \ldots$ the ground-state Eq. (9) corresponds to the fractional Hall effect, $\nu = 1/m$. As the activation gap is experimentally observable from the activation behavior of the direct current magneto-transport coefficients, it is given by the excitation gap of relevant compound exciton, at $m \geq 3$, and by the gap of pertinent compound spin-exciton, at $m = 1$.

Point out that present study has some important similarities with well known theory by Yoshioka, Halperin and Lee [4] of ground-state of the $\nu = 1/3$ fractional quantum Hall effect (for related recent works see, e.g., [5]), using periodic boundary condition along x and y. In particular, the Hamiltonian of [4], for n electrons in the rectangular cell $b \times a$, can be related with the Hamiltonian, Eq. (4), $\hat{H}_{\tilde{N}}(\mathbf{r}_1, \ldots, \mathbf{r}_{\tilde{N}})$ for $\tilde{N} \equiv n \to \infty$ electrons within MS. If to take into account that the role of b (a) now is played by $L_x^{\square}$ (L_y), etc.; some minor differences are related with another form of the Landau gauge in Ref. [4]. Present study shows that proper periodic boundary condition can be totally relevant to symmetry, periodicity, correlations, and etc. properties of a sought state; i.e., it will not lead to any oversimplification.

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